Prediction of Elastic Modulus of Concrete Using Support Vector Committee Method

Javad Sadoghi Yazdi[1]; Farzin Kalantary[2]; and Hadi Sadoghi Yazdi[3]

Abstract: Knowledge about concrete properties is of utmost importance in engineering materials, and elastic modulus is one of concrete's most important properties that is used in the calculation of deformation of structures. For this reason, many researchers have attempted to introduce various correlations between this property and the compressive strength. In this paper, support vector committee (SVC) is used for prediction of elastic modulus of normal strength (NSC) and high-strength concrete (HSC). The SVC is based on learning theory, and deploys the technique by introducing accuracy insensitive loss function. The comparison between concrete elastic modulus predicted by the SVC method with the experimental data and those from other methods like support vector machine (SVM), artificial neural networks (ANN), fuzzy logic, and other conventional methods show marked improvement in relation to the best of prediction methods with error indices constantly less than 1%. It is therefore concluded that the SVC model is a greatly more effective method of prediction for elastic modulus of all grades of concrete. **DOI: 10.1061/(ASCE)MT.1943-5533.0000507.** *© 2013 American Society of Civil Engineers.*

CE Database subject headings: Elasticity; Predictions; Compressive strength; Concrete.

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Introduction

Concrete is perhaps the most important construction material. Important features of concrete, such as excellent resistance to water, ability to form virtually any shape and size, and relatively low production cost, have led to increased use of concrete in one form or another for almost all structures, great or small (e.g., buildings, bridges, pavement, dams, reactor vessels, retaining walls, tunnels, drainage and irrigation facilities, and tanks) [American Concrete Institute (ACI) Committee 318 1995]. Therefore, knowledge of the behavior of concrete is one of the most important issues in civil engineering.

Elastic modulus of concrete is one of the most important mechanical properties of this crucial material. Knowledge of the modulus of elasticity is essential for the designer in estimating the deformation of structural elements under service conditions in reinforced and prestressed concrete and in mass concreting.

Many authors have emphasized the importance of elastic modulus of concrete; over the past 25 years, numerous researches have focused on the development of the relationship between elastic modulus and compressive strength of concrete. Baalbaki et al. (1992) predicted elastic modulus for high-strength concrete (HSC). Mesbah et al. (2002) determined elastic properties of high-performance concrete, Wee et al. (1996) determined the stress–strain relationship of HSC in compression, Turan and Iren (1997) introduced strain–stress relationship of concrete, Demir (2005) predicted elastic modulus of normal and high-strength concrete by fuzzy logic and then predicted it by artificial neural networks (Demir 2008). Finally, Yan and Shi (2010) predicted elastic modulus of normal and high-strength concrete by support vector machine.

Background Knowledge

Multiple Classifier Systems

Ensemble systems, also called committees or multiple systems, offer a solution to regression/classification problems. The idea of combining multiple systems or classifiers is based on the observation that achieving optimal performance in combination is not necessarily consistent with obtaining the best performance for a base system/classifier. The rationale is that it may be easier to optimize the design of a combination of relatively simple systems than to optimize the design of a single complex system. The increase in accuracy by using multiple systems is at least partially a result of diversity (Kuncheva and Whitaker 2003). Diverse systems include a committee of various experts for obtaining better performance.

Multiple classifier systems (MCS) can be categorized from different points of view. One major categorization is based on the capability of being trained, as shown in Fig. 1.

Nontrainable combiners do not need to be trained after the classifiers in the ensemble have been trained individually. Trainable combiners need additional training after complete training of base classifiers. The third class of ensemble systems is those that develop the combiner during training of individual classifiers. Some known trainable combiners are weighted average, fuzzy integral, decision template, and Dempster Shafer. The most known nontrainable combiners are min, max, median, simple mean, product, and majority vote.

The main motivation in this paper is based on clustering of input space and appropriating one expert to each subspace with fuzzy

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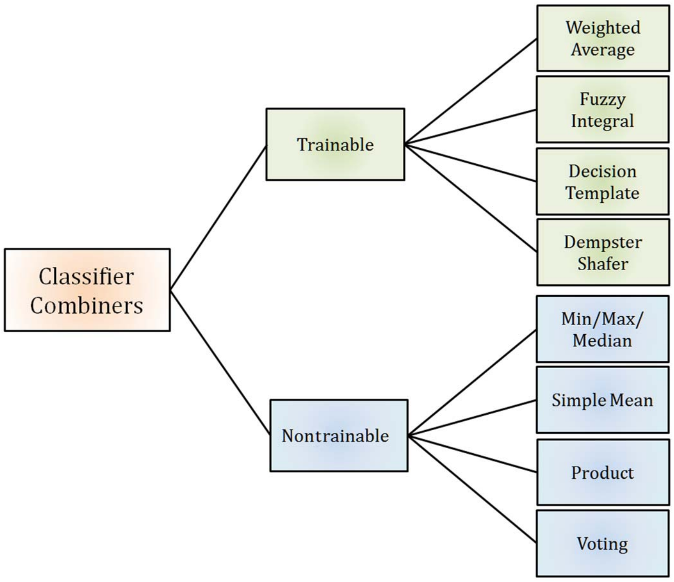

Fig. 1. Hierarchy of MCS fusion methods

relation. Input space is divided to several subspaces, and in each subspace, a support vector regression (SVR) models the subspace data. A weighting procedure is performed using a probability density function of each subspace and gives a portion of each SVR according to generated rules. Then results of weighted SVRs are combined, and fitting is performed.

Review of Static Modulus of Elasticity

Although it is a very simple test and a matter of routine procedure to determine concrete characteristic strength, evaluation of the modulus of elasticity is a rather tedious task. Nevertheless, the elastic modulus of concrete is one of the most important parameters in design and safety evaluation for most structures. The static modulus of elasticity for a material under tension or compression is given by the slope of the stress–strain curve for concrete under uniaxial loading. A full description of various parameters that influence the modulus of elasticity of concrete is presented by Mehta and Monteiro (2006). Ideally, elastic modulus is measured directly on concrete samples under compression by recording the load–deformation curve. However, this is not always easy from an experimental point of view. This testing procedure is much more complicated and time-consuming when compared to compressive strength tests carried out to obtain the compressive strength, f_c.

There is a general agreement that the modulus of elasticity increases with the increase in compressive strength of concrete. Therefore, various national building codes have proposed a number of formulas for normal strength concrete (NSC) and HSC.

Relationships for NSC are given as follows:

ACI 318-95 (ACI Committee 318 1995):

$$E_c = 4.73(f_c)^{1/2} \tag{1}$$

TS-500 (Turkish Standardization Institute 2000):

$$E_c = 3.25(f_c)^{1/2} + 14 \tag{2}$$

Relationships f_c and E_c are expressed in MPa and GPa, respectively. Concrete with compressive strengths exceeding 41.37 MPa are referred to as high-strength concretes. High-strength concretes are sometimes used for both precast and prestressed members. American, European, and Norwegian committees on high-strength concrete propose the following relationships for HSC:

ACI 363 (ACI Committee 363 1984):

$$E_c = 3.32(f_c)^{1/2} + 6.9 \tag{3}$$

CEB90 (Comité Euro-International du Béton-Fédération Internationale de la Précontrainte (CEB-FIP) Model Code 1993):

$$E_c = 10(f_c + 8)^{1/3} \tag{4}$$

NS 3473 (Norwegian Council for Building Standardization 1992):

$$E_c = 9.5(f_c)^{0.3} \tag{5}$$

Review of Support Vector Regression

The SVR is a statistical learning method that generates input-output mapping functions from a set of training data.

Support vector machines (SVM) were originally introduced by Vapnik (1995) within the area of statistical learning theory and structural risk minimization, and create a classifier with minimized Vapnik–Chervonenkis (VC) dimension. SVR is a supervised learning method that generates input-output mapping functions from a set of labeled training data. The mapping function can be either a classification function, i.e., the category of the input data, or a regression function.

Suppose training data $\{(X_1, Y_1), (X_2, Y_2), \ldots, (X_l, Y_l)\} \subset X \times R$ are given, where X denotes the space of the input patterns (e.g., $X = R^D$). ε-SVR is the most used method for support vector regression up to now. For the purpose of estimating the regression curves of nonlinear functions, Vapnik (1995) introduced ε-insensitive function as the loss function, in which the selection of design data C and ε are very important to construct the regression functions. The data ε indicate the error expectation (requirement to error) of the system on the estimation function in the sample data point. The data C is the penalty for the sample data with its estimation function error larger than ε.

In ε-SV regression (Vapnik 1995), the goal is to find a function $f(x)$ that has at the most ε deviation from the actually obtained targets (y_i) for all the training data. The regressor must not only fit the given data well, but also make minimal errors in predicting the values at any other arbitrary point in the R^D. Nonlinear regression is accomplished by fitting a linear regressor in a higher dimensional feature space. A nonlinear transformation ϕ is used to transform data points from the input space (with dimension D) into a feature space having a higher dimension L ($L > D$), as shown in Fig. 2. The nonlinear mapping is denoted by $\phi{:}R^D \to R^L$

This problem can be written as a convex optimization problem, hence (Vapnik 1995)

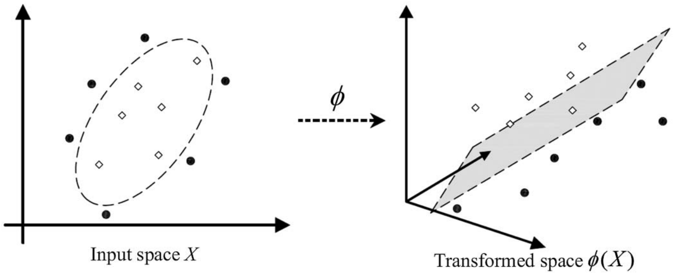

Fig. 2. Mapping of data set X by ϕ into a higher dimensional space

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$$\begin{aligned}&\text{Min}\ \frac{1}{2}\|W\|^2 + C\left(\sum_{i=1}^{l}(\xi_i+\xi_i^*)\right) \quad s.t.\, y_i - W^T\phi(X_i) - b \le \varepsilon + \xi_i\\ &-y_i + W^T\phi(X_i) + b \le \varepsilon + \xi_i^* \qquad \xi_i, \xi_i^* \ge 0 \quad \forall\ i = 1, \ldots, l\end{aligned} \tag{6}$$

where $C > 0$ = constant; and ξ_i, ξ_i^* = slack variables for soft margin SVR, which allow accepting some deviation larger than ε for precision.

In most cases, the optimization problem, Eq. (6), can be solved more easily in its dual formulation

$$\begin{aligned}&\text{Max}\ -\frac{1}{2}\sum_{i,j=1}^{l}(\alpha_i-\alpha_i^*)(\alpha_j-\alpha_j^*)\,K(X_i,X_j) - \varepsilon\sum_{i=1}^{l}(\alpha_i+\alpha_i^*)\\ &+\sum_{i=1}^{l}y_i(\alpha_i-\alpha_i^*) \qquad s.t\sum_{i=1}^{l}(\alpha_i-\alpha_i^*)=0, \qquad \alpha_i,\alpha_i^*\in[0,C]\end{aligned} \tag{7}$$

where α_i, α_i^* = Lagrange coefficients; and matrix K = kernel matrix with its elements given by $K(X_i, X_j) = \phi(X_i)^T\phi(X_j)$, $i, j = 1, 2, \ldots, M$.

By solving Eq. (7) the Lagrange coefficients can be found, and replacing them, yields

$$\begin{aligned}W &= \sum_{i=1}^{l}(\alpha_i-\alpha_i^*)\times\phi(X_i), \quad \text{thus}\\ f(x) &= \sum_{i=1}^{l}(\alpha_i-\alpha_i^*)K(x,X_i)+b\end{aligned} \tag{8}$$

SVR can use arbitrary kernel function, but for most application problems, including this work, several kernel functions in common use, such as exponential radial basis function (erbf) kernel, radial basis function (rbf) kernel, and polynomial kernel, are chosen. The erbf kernel function is $\exp(-\sqrt{\|x-c\|^2}/2\sigma^2)$, where c is a constant and σ is the width of Gaussian. The rbf kernel function is $\exp(-\sqrt{\|x-c\|^2}/2\sigma^2)$, where c is a constant and σ is the width of Gaussian. The polynomial kernel function is $(\|x\|^2+1)^p$, where p is the degree of polynomial function (Haykin 1999).

Support Vector Committee

As previously mentioned, the support vector machine is an approximate implementation of the method of structural risk minimization. Some problems in the SVR are as follows:

1. Because each sample is one constraint in the support vector, increasing training samples is equivalent to increasing the

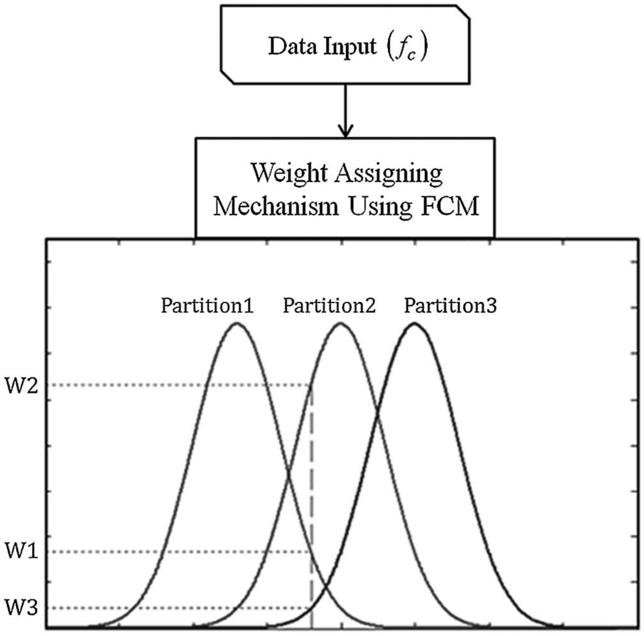

Fig. 3. Assigning weights to the input data

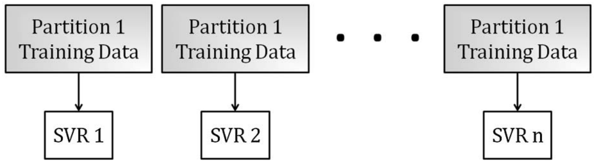

Fig. 4. Applying SVR in each partition

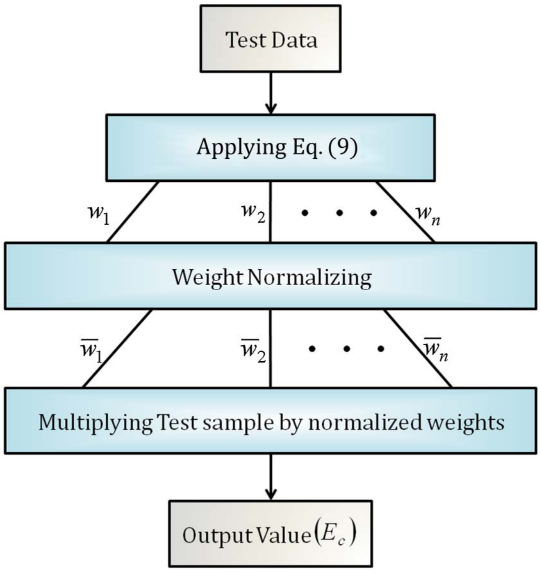

Fig. 5. Testing procedure

Table 1. Statistics of Parameters for the Training and Testing Subsets Used in Development of the SVC Model

		Input and output parameters			
		NSC		HSC	
Statistical parameters	Subset	f_c (Mpa)	E_c (GPa)	f_c (Mpa)	E_c (GPa)
Max	Training	47.70	36.80	125.60	53.20
	Testing	37.50	32.60	100.60	50.50
Min	Training	14.00	15.60	46.40	35.20
	Testing	16.20	18.00	57.90	40.80
Mean	Training	27.61	28.12	85.48	45.82
	Testing	25.28	26.02	82.34	44.77
Standard deviation	Training	6.32	4.00	13.83	2.71
	Testing	5.38	3.86	11.98	2.53

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Table 2. Average RMSE and MAPE of SVC Method for Training and Testing Data from NSC

		Average MAPE (%)			Average RMSE		
Kernel	Data set	Cluster = 2	Cluster = 3	Cluster = 4	Cluster = 2	Cluster = 3	Cluster = 4
erbf	Train	0.869	0.592	1.420	0.264	0.202	0.437
erbf	Test	1.148	0.912	1.942	0.400	0.266	0.521
rbf	Train	1.242	4.367	1.347	0.357	0.729	0.530
rbf	Test	1.648	6.934	0.894	0.452	0.709	0.288
poly ($p = 2$)	Train	1.774	5.452	7.127	0.500	1.602	2.320
poly ($p = 2$)	Test	1.924	4.453	7.634	0.518	1.123	1.998
poly ($p = 3$)	Train	0.794	2.079	1.884	0.246	0.627	0.626
poly ($p = 3$)	Test	1.012	1.235	3.726	0.286	0.347	0.944
poly ($p = 4$)	Train	2.724	1.773	2.793	0.800	0.554	0.872
poly ($p = 4$)	Test	1.977	0.874	0.878	0.602	0.287	0.279

Table 3. Average RMSE and MAPE of SVC Method for Training and Testing Data from HSC

		Average MAPE (%)			Average RMSE		
Kernel	Data set	Cluster = 2	Cluster = 3	Cluster = 4	Cluster = 2	Cluster = 3	Cluster = 4
erbf	Train	0.238	0.423	0.847	0.147	0.221	0.412
erbf	Test	0.615	0.933	1.250	0.324	0.463	0.598
rbf	Train	0.425	5.112	1.444	0.226	2.413	0.674
rbf	Test	0.505	5.443	1.794	0.278	2.459	0.828
poly ($p = 2$)	Train	1.290	1.019	0.431	0.636	0.496	0.254
poly ($p = 2$)	Test	1.061	0.711	0.527	0.548	0.393	0.289
poly ($p = 3$)	Train	1.120	1.660	1.852	0.533	0.804	0.868
poly ($p = 3$)	Test	0.677	1.459	1.456	0.374	0.715	0.715
poly ($p = 4$)	Train	0.305	1.372	3.150	0.170	0.682	1.456
poly ($p = 4$)	Test	0.224	1.184	2.775	0.419	0.601	1.280

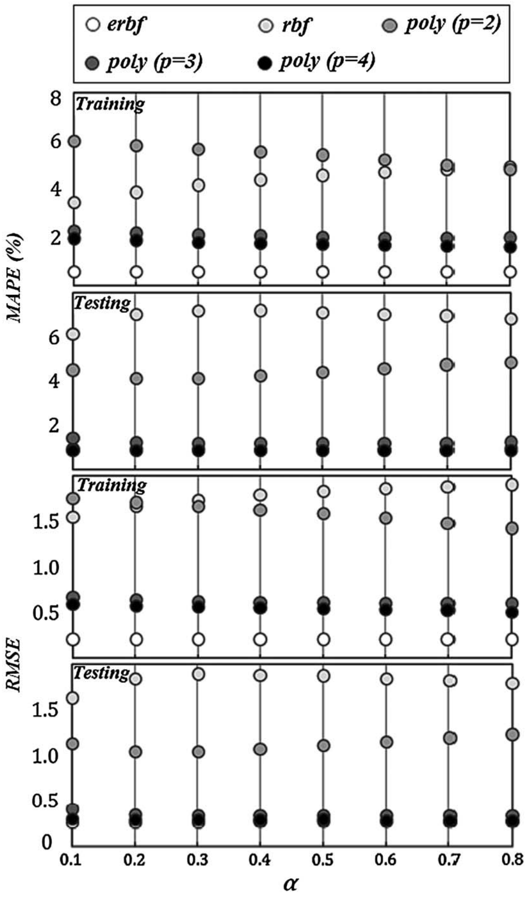

Fig. 6. Sensitivity study of α parameter and kernel (erbf, rbf, and polynomial 2, 3, and 4) on MAPE and RMSE for NSC per cluster 3

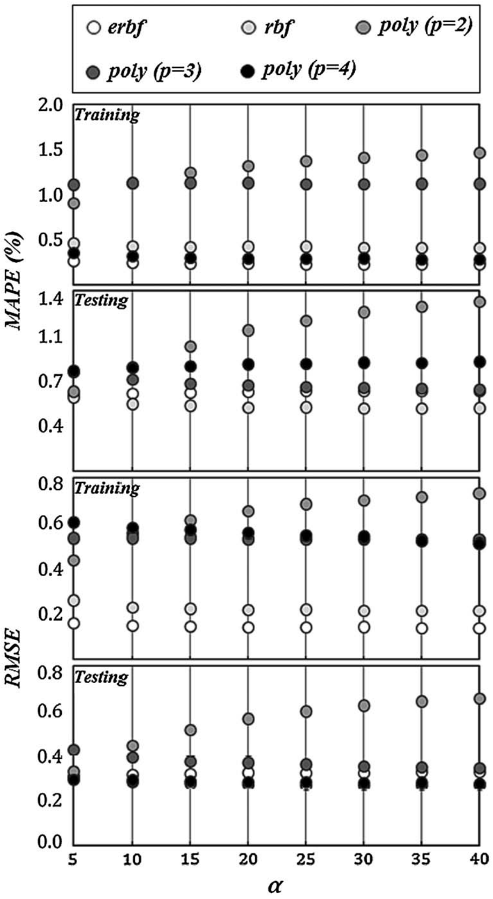

Fig. 7. Sensitivity study of α parameter and kernel (erbf, rbf, and polynomial 2, 3, and 4) on MAPE and RMSE for HSC per cluster 2

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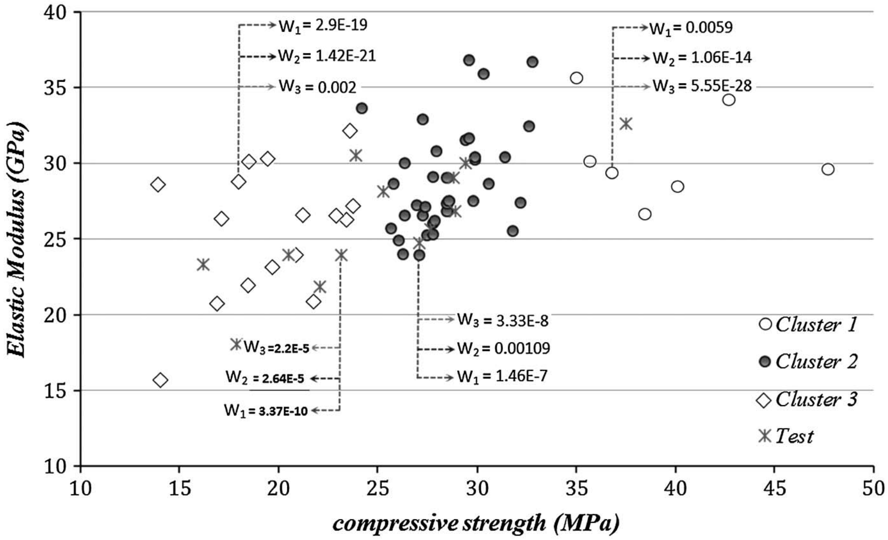

Fig. 8. NSC data clustering

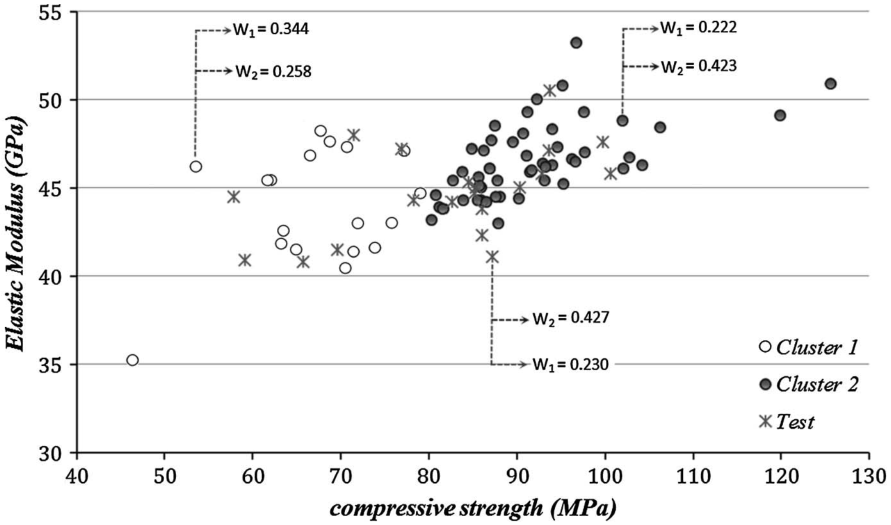

Fig. 9. HSC data clustering

number of constraints. In this case, solving equations to find the optimal hyperplane becomes very hard.

2. Finding a suitable kernel for modeling of nonlinear space.

SVC is based on the divide and conquer principle, which can solve the two aforementioned problems. Input space is divided into several subspaces, and in each subspace a SVR model of the data is made. This causes the new generated space to have the properties of high-dimensional space. A weighting procedure is performed using a probability density function on each subspace and gives a portion of each SVR model according to the generated rules. Then, results of weighted SVRs are combined and fitting is performed. In other words, instead of using a single SVR model over the whole space, SVR is applied to subspaces.

The SVC method is described in detail in the following steps (Huang and Shen 2008):

Step 1: First, input training data is divided into n subsets. An algorithm of clustering, like fuzzy C-means (FCM), can be used for this grouping (see more details in the appendix). Indeed, the weights assigned to any input data using FCM (Fig. 3) shows three partitions (for example) that FCM has clustered. The probability density function (PDF) of each cluster is obtained, as shown in the following figure, then the corresponding weights of the input data are calculated based on membership values to each partition (i.e., clusters).

Step 2: Each available subset for any partition is applied for training of each SVR (see Fig. 4). For training samples

Table 4. Statistical Parameters of Membership Values for Each Test Sample

		Membership values for each test sample (W_i)				
		HSC		NSC		
Statistical parameters	Subset	W_1	W_2	W_1	W_2	W_3
Max	Training	0.372	0.439	1.03E-02	3.72E-01	4.39E-01
	Testing	0.37198	0.43928	0.01033	0.37198	0.43928
Min	Training	0.08637	0.20702	5.25E-26	4.14E-64	1.22E-70
	Testing	0.08637	0.20702	0.01033	0.08602	0.07039
Mean	Training	0.30457	0.40724	1.59E-03	3.25E-02	4.18E-02
	Testing	0.2511	0.37526	0.00689	0.21992	0.2815

Table 5. Comparison of Errors Estimated by SVC and Other Models for Training Data from NSC

f_c (MPa)	E_c (GPa)	ACI 318	TS 500	Regression (Demir 2005)	Fuzzy (Demir 2005)	ANN (Demir 2008)	SVM (Yan and Shi 2010)	SVC
31.4	30.4	−4	1.8	−0.6	−0.3	−0.9	−0.6	0.086
27.8	29.1	−4.1	2	−0.9	−0.9	−0.9	−0.3	−0.040
28.5	26.8	−1.6	4.6	1.9	1.9	1.6	2.4	0.091
29.4	31.5	−6	0	−2.5	−2.5	−2.8	−2.2	−0.084
26.4	30	−5.7	0.6	−2.4	−2.4	−2.4	−2.1	−0.018
28.5	29	−3.8	2.3	−0.3	−0.3	−0.6	0	0.024
32.6	32.4	−5.5	0	−1.9	−1.6	−2.3	−2.6	−0.130
29.9	30.2	−4.2	1.5	−0.9	−0.9	−1.2	−0.6	−0.015
29.8	27.5	−1.7	4.1	1.7	1.7	1.4	1.9	0.121
28	30.8	−5.9	0.3	−2.5	−2.5	−2.5	−1.8	−0.042
27.3	26.5	−1.9	4.5	1.6	1.6	1.3	2.1	0.156
27.5	25.2	−0.5	5.8	3	3	2.8	3.3	0.208
27	27.2	−2.7	3.8	0.8	0.8	0.5	1.1	0.108
28.5	27.3	−1.9	4.1	1.4	1.4	1.1	1.9	0.101
26.4	26.5	−2.1	4.2	1.3	1.3	1.1	1.6	0.138
27.1	23.9	0.7	6.9	4.1	4.1	4.1	4.5	0.256
26.3	24	0.2	6.7	3.6	3.6	3.6	4.1	0.250
26.1	24.9	−0.7	5.7	2.7	2.5	2.5	3	0.206
27.8	25.3	−0.3	5.8	3	3	2.8	3.5	0.182
25.7	25.7	−1.8	4.9	1.8	1.5	1.5	1.8	0.163
27.8	26	−1	5.2	2.3	2.3	2.1	2.6	0.145
28.6	27.5	−2.2	3.9	1.1	1.4	1.1	1.4	0.075
27.9	26.2	−1	5	2.1	2.4	2.1	2.6	0.134
18.4	21.9	−1.8	5.9	1.8	2	3.1	−1.5	0.309
23.4	26.3	−3.4	3.4	0	0	0.3	−0.5	0.100
29.9	30.4	−4.6	1.5	−1.2	−0.9	−1.5	−0.9	−0.088
22.9	26.5	−4	2.9	−0.5	−0.5	−0.3	−1.3	0.093
23.7	27.2	−4.1	2.7	−0.8	−0.8	−0.5	−1.1	0.060
27.4	27.1	−2.4	3.8	1.1	1.1	0.8	1.6	0.064
14	15.6	0.2	10.9	5.5	4.2	8.3	−0.5	0.606
16.9	20.5	−1	7	2.5	1.2	4.1	−1.8	0.367
17.1	26.3	−6.8	1.1	−3.2	−4.5	−1.6	−5.3	0.088
18	28.8	−8.6	−1.2	−5.2	−6.3	−4	−4.6	−0.034
18.5	30.1	−9.6	−0.21	−6.3	−7.2	−5.1	−5.4	−0.099
21.8	20.9	1.3	8.4	4.6	4	5	3.3	0.315
25.8	28.6	−4.6	2	−1.1	−1.1	−1.1	−0.9	−0.048
27.3	32.9	−8.2	−2	−4.9	−4.9	−4.9	−4.3	−0.249
30.3	35.9	−9.7	−3.9	−6.5	−6.5	−6.8	−6.1	−0.387
29.6	36.8	−11	−5.2	−6.7	−7.7	−8.1	−7.4	−0.422
19.6	23.1	−2.1	5.3	1.2	0.2	2.3	−1.4	0.203
19.4	30.3	−9.4	−1.8	−6.1	−7	−4.8	−7.6	−0.133
20.9	23.9	−2.2	5	1.2	0.5	1.9	−0.7	0.154
21.2	26.5	−4.8	2.4	−1.3	−2.1	−0.8	−2.9	0.025
23.6	32.1	−9	−2.2	−5.8	−6.1	−5.5	−6.1	−0.239
24.2	33.6	−10.4	−3.7	−7.1	−7.1	−6.7	−5	−0.309
31.8	25.5	1	6.9	4.3	4.6	4.3	4.3	0.073
32.2	27.4	−0.5	4.9	2.7	3	2.5	2.5	−0.015
30.6	28.6	−2.3	3.4	0.9	0.9	0.9	1.1	−0.071
29.6	31.6	−6	0	−2.5	−2.5	−2.8	−2.2	−0.206
35	35.6	−7.5	−2.5	−4.3	−3.9	−4.3	−6.1	−0.383
32.8	36.7	−9.5	−4	−6.2	−6.2	−6.6	−7	−0.433
38.4	26.6	2.7	7.7	5.9	6.9	6.4	1.3	0.038
35.7	30.1	−1.8	3.3	1.5	2.1	1.5	−0.9	−0.122
42.7	34.1	−3.1	1	0	1.7	1	−4.4	−0.295
36.8	29.3	−0.6	4.4	2.6	3.2	2.9	−0.6	−0.064
40.1	28.4	1.4	6.2	4.8	6	5.4	−1.1	−0.019
47.7	29.6	3	6.8	5.9	8.9	7.7	−4.4	−0.076

of partition 1 (as shown in Fig. 3), SVR1 is trained as shown in Fig. 4.

In this paper, some kernel functions, such as polynomials and radial basis functions, have been used for SVR to obtain the best state.

Step 3: The third step is the testing procedure. The hold out method is used for computing average error of the proposed method and comparing it with other methods. To calculate the output of the proposed system, membership values for each test sample (W_i) should be computed. By applying Eq. (9) for any test sample, corresponding weights to the test samples are obtained, and then these weights are normalized by dividing any weight to the sum of them. At the end, these normalized

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Table 6. Comparison of Errors Estimated by SVC and Other Models for Training Data from HSC

f_c (MPa)	E_c (GPa)	ACI 363	CBE	NS 3473	Wee et al. 1996	Gesoglu et al. 2002	Regression (Demir 2005)	Fuzzy (Demir 2005)	ANN (Demir 2008)	SVM (Yan and Shi 2010)	SVC
63.2	41.8	−8.4	−0.4	−8.8	−1.3	−3.3	−1.7	0.4	0.8	2.9	0.199
72	43	−8.2	−0.4	−9	−0.9	−2.2	−0.9	0	0.9	0	0.116
65.1	41.5	−7.9	0.4	−8.3	−0.4	−2.5	−0.8	0.8	1.2	2.5	0.203
70.5	40.4	−5.7	2.4	−6.5	1.6	0.4	1.6	2.8	3.6	2.4	0.263
71.5	41.4	−6.6	1.7	−7	0.8	0	0.8	2.1	2.9	1.7	0.202
63.6	42.6	−9.4	−1.3	−9.4	−1.7	−3.8	−2.1	−0.4	0	2.1	0.138
85.9	45	−7.2	0.5	−9	0	0.9	0.9	0.5	0.5	0	0.012
90.2	44.4	−5.8	1.8	−7.5	1.3	2.7	2.2	1.8	1.3	1.3	0.052
85.9	44.3	−6.6	1.3	−8	0.9	1.3	1.3	1.3	1.3	0.4	0.062
81.2	43.9	−7	0.9	−8.3	0.4	0.4	0.9	0.9	1.3	0	0.085
88.1	44.5	−6.2	1.3	−8	0.9	1.8	1.8	1.3	1.3	0.9	0.054
81.6	43.8	−7	0.9	−8.3	0.4	0.4	0.9	1.3	1.3	0	0.094
84.8	47.2	−9.9	−1.9	−11.3	−2.4	−1.9	−1.4	−1.9	−1.9	−2.4	−0.092
85.6	45.6	−8.2	0	−9.6	−0.5	0	0	0	0	−0.9	−0.002
96.2	46.6	−7	0.5	−9.3	0	2.3	1.4	0.5	0.9	0	−0.053
46.4	35.2	−5.6	2.8	−5.3	1.4	−3.2	0	2.8	1.4	0	0.572
73.9	41.6	−6.2	1.7	−7.1	1.2	0.4	1.2	2.1	2.9	1.2	0.213
87.6	44.5	−6.7	1.3	−8	0.9	1.8	1.8	1.3	1.3	0.9	0.054
93.1	45.4	−6.4	1.4	−8.2	0.9	2.3	1.8	1.4	2.3	0.9	0.007
95.3	45.2	−5.9	1.8	−8.1	1.4	3.2	2.7	1.8	2.3	1.4	0.021
102.1	46.1	−5.5	1.8	−7.8	1.4	4.1	3.2	2.3	1.4	0.9	−0.026
102.8	46.7	−6.1	1.4	−8.4	0.9	3.7	2.8	1.9	0.9	0.5	−0.058
106.3	48.4	−7.3	0	−9.7	0	2.9	1.9	0.5	−0.5	−1.5	−0.150
104.2	46.3	−5.6	1.9	−7.9	1.9	4.6	3.2	2.3	1.4	0.9	−0.031
94.6	47.3	−8	−0.5	−9.9	−0.9	0.9	0.5	−0.5	0	−0.9	−0.084
94	46.3	−7.4	0.5	−9.3	0	1.9	1.4	0.5	1.4	0	−0.027
96.6	46.5	−7	0.5	−9.3	0.5	2.3	1.9	0.9	0.9	0	−0.036
91.5	45.9	−7.3	0.5	−9.2	0	1.4	1.4	0.5	1.4	0	−0.001
91.7	46	−7.4	0.5	−9.2	0	1.4	0.9	0.5	1.4	0	−0.005
119.9	49.1	−5.9	1.5	−9.3	1	5.9	3.9	2	0	0.5	−0.175
125.6	50.9	−6.6	0	−10.2	0	5.6	3.1	1	0	0	−0.273
77.2	47.1	−10.8	−3.3	−12.2	−4.2	−3.8	−3.3	2.8	−2.4	−3.8	−0.061
66.5	46.8	−12.6	−4.7	−13.1	0.7	−5.6	−5.6	−4.2	−3.7	−3.3	−0.043
70.7	47.3	−12.3	−4.3	−13.2	−6.1	−5.2	−5.2	−4.3	−3.3	−4.3	−0.072
61.8	45.4	−12.3	−4.1	−12.7	−7.3	−5	−5.4	−3.6	−3.2	0	0.034
68.9	47.6	−12.3	−5.2	−13.8	−7.1	−5.7	−5.7	−4.8	−3.8	−4.3	−0.087
62.2	45.4	−12.3	−4.1	−12.7	−7.3	−5	−5.4	−3.6	−3.2	0	0.036
75.8	43	−7.3	0.9	−8.2	−0.4	0	0.4	0.9	1.7	0	0.168
67.7	48.2	−14	−5.8	−14.5	−1.2	−6.7	−6.7	−5.3	−4.8	−4.8	−0.119
53.6	46.2	−14.8	−6.9	−14.8	−11.1	−7.9	−8.8	−5.5	−6.5	0	−0.007
92.9	46.4	−7.4	0	−9.3	1.4	0	0.9	0.5	0.9	0	−0.018
94	48.3	−9.2	−1.4	−11.1	0	−1.9	−0.5	−1.4	−1	−1.9	−0.123
97.7	47	−7.1	0.5	−9.4	2.4	0	1.4	0.5	0.5	0	−0.049
102	48.8	−8.3	−1	−10.7	1.5	−1	0.5	−0.5	−1	−2	−0.149
86.2	47.1	−9.4	−1.4	−10.8	−1.4	−1.9	−1.4	−1.4	−1.4	−2.4	−0.054
87.9	43	−5.2	2.6	−6.5	3.4	2.2	3.4	3	2.6	2	0.173
82.7	45.4	−8.2	−0.5	−9.5	−0.5	−0.9	−0.5	−0.5	0	−1.4	0.039
79.1	44.7	−8.5	−0.4	−9.4	−0.9	−0.9	−0.4	0	0.4	−0.9	0.078
86.9	46.1	−8.3	−0.5	−9.7	0	−0.9	0	−0.5	−0.5	0.5	0.000
85.5	44.3	−6.6	0.9	−8.4	1.3	0.4	1.3	1.3	1.3	−5.3	0.100
91.1	46.8	−8.4	−0.5	−9.8	0.5	−0.9	0	−0.5	0.5	−0.9	−0.039
96.7	53.2	−13.8	−5.9	−16	−4.3	−6.4	−4.8	−5.9	−5.9	6.4	−0.393
91.2	49.3	−10.8	−3	−12.3	−2	−3.5	−2.5	−3	−2	−3.5	−0.176
83.8	45.9	−8.7	−0.9	−10.1	−0.9	−1.4	−0.5	−0.5	−0.5	−1.4	0.013
87.1	47.7	−10	−1.9	−11.4	−1.4	−2.4	−1.4	−1.9	−1.9	−2.4	−0.087
93.2	46.2	−7.4	0.5	−9.2	1.8	0	1.4	0.5	1.4	0	−0.003
86.9	46.1	−8.3	−0.5	−9.7	0	−0.9	0	−0.5	−0.5	−0.9	0.002
90.7	48.1	−9.6	−1.9	−11.5	−1	−2.4	−1	−1.9	−1	−2.4	−0.108
89.5	47.6	−9.5	−1.4	−10.9	−1	−1.9	−1	−1.4	−1.9	−1.9	−0.080
87.8	45.4	−7.3	0.5	−9.1	0.9	0	0.9	0.5	0.5	0	0.042
95.2	50.8	−11.7	−4.1	−13.7	−2.5	−4.1	−3	−3.6	−3	−4.1	−0.258
92.2	50	−11	−3.5	−13	−2.5	−4	−3	−3.5	−2.5	−4.5	−0.213
97.6	49.3	−9.4	−2	−11.8	0	−2.5	−1	−2	−1.5	−2.5	−0.173
87.5	48.5	−10.7	−2.9	−12.1	−2.4	−3.4	−2.4	−2.4	−2.9	−3.4	−0.129
80.4	43.2	−6.5	1.3	−7.8	0.9	0.9	1.3	1.7	1.7	0.4	0.165
86.5	44.2	−6.6	1.3	−8	1.8	0.9	1.8	1.3	1.3	0.9	0.109

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Table 6. (*Continued*)

f_c (MPa)	E_c (GPa)	ACI 363	CBE	NS 3473	Wee et al. 1996	Gesoglu et al. 2002	Regression (Demir 2005)	Fuzzy (Demir 2005)	ANN (Demir 2008)	SVM (Yan and Shi 2010)	SVC
83.9	44.3	−7.1	0.9	−8.4	0.9	0.4	0.9	0.9	1.3	0	0.104
80.9	44.6	−8	0	−8.9	−0.4	−0.4	0	0.4	0.4	−0.9	0.086
85.7	45.1	−7.7	0.5	−9	0.5	0	0.9	0.5	0.5	−0.5	0.058

weights are multiplied by each test sample to generate final values. Fig. 4 shows this procedure.

$$w_i = \frac{1}{(2\pi)^{d/2}|\tilde{\Sigma}|^{1/2}} \exp\left[-\frac{1}{2}(x-\mu)^t\tilde{\Sigma}^{-1}(x-\mu)\right] \qquad (9)$$

where $\tilde{\Sigma} = \alpha\Sigma$ and Σ = covariance samples; μ = mean value of training data; d = length of feature vector; α = variable to control spreading of each Gaussian distribution that is considered for samples of the training set; x_t = test sample; and $|\cdot|$ = determinant.

The output value in Fig. 5 is the computed value for elastic modulus of the test sample. To compute error of the hold out method for evaluation of performance of the proposed method, root mean square error (RSME) and mean absolute percentage error (MAPE) are used.

Support Vector Committee for Prediction of Elastic Modulus of NSC and HSC

As previously mentioned, many methods have been proposed for the prediction of elastic modulus of concrete from its compressive strength. This study attempts to use support vector committee (SVC) for the prediction of elastic modulus. The experimental results from Wee et al. 1996; Gesoglu et al. 2002, and Yan and Shi 2010 for HSC (Ozturan 1984; Turan and Iren 1997) and for NSC (Yan and Shi 2010) are used in this study. For the SVC, the inputs are f_c, and the output is the measured elastic modulus. The training and test data sets used in this paper have been obtained from previous experimental works (Demir 2005). From a total of 70 and 89 case histories of recorded NSC and HSC, respectively, 57 and 69 cases were used for training of the SVC models, whereas the remaining 13 and 20 cases were used for testing of the trained SVC models. The statistical parameter is shown in Table 1.

Numerical investigation was carried out and the fuzzy results were compared with those of test data and some other available literature. Numerical results revealed a good agreement between the test and fuzzy results. To have an objective comparison of the performance of the models against the experimental results, the error measured by the root mean square error ($\mathrm{RMSE} = \sqrt{(\sum_{i=1}^{n} \mathrm{err}_i^2)/n}$) and mean absolute percentage error ($\mathrm{MAPE} = \sum_{i=1}^{n} |\mathrm{err}_i/E_c| \times 100$) were computed for each model, where err is the difference between predicted and experimented values, and n is the number of data sets.

SVC Parameters Sensitivity

In this section, the concrete elastic modulus is calculated by the proposed SVC. The cluster and α parameters greatly affect the results of the SVC model. The values of the cluster, α, and kernel types were chosen by a trial-and-error approach. The hold out method (separate training and testing data sets are used) was used for testing the proposed method with different kernel functions for SVC and different values for α (0.1, 0.2, …, 0.8 for NSC and 5, 10, …, 40 for HSC) and clusters (that considered 2, 3, and 4).

In the training process, RMSE and the MAPE were used as the main criteria to evaluate the performance of the SVC model.

Kernels are discussed in the literature, but it is important to choose one that gives the best generalization with a given data set. Because the choice of kernel may affect the prediction capacities of the SVC's RMSE and MAPE, values were compared to appraise a suitable choice of kernels. Three types of kernel functions have proven to be better than other kernels, namely erbf, rbf, and the polynomial function for NSC and HSC.

Average RMSE and MAPE (%) of the SVC method for training and testing data from NSC and HSC per different cluster and kernel functions is shown in Tables 2 and 3. The erbf kernel function with cluster 3 for NSC data and the erbf kernel function with cluster 2 for

Table 7. Comparison of Errors Estimated by SVC and Other Models for Testing Data from NSC

f_c (MPa)	E_c (GPa)	ACI 318	TS 500	Regression (Demir 2005)	Fuzzy (Demir 2005)	ANN (Demir 2008)	SVM (Yan and Shi 2010)	SVC
29.4	30	−7.3	−1.3	−4	−4	−4.3	−3.3	−0.231
28.8	29	−3.5	2.3	−0.3	−0.3	−0.6	0.3	−0.160
27.7	25.6	−0.8	5.4	2.6	2.6	2.6	3.1	0.074
22.1	21.8	0.4	7.4	3.9	3.7	4.4	2.6	0.333
28.9	26.8	−1.3	4.6	1.9	2.1	1.9	2.4	−0.006
20.6	23.9	−2.4	4.8	1	1	1.7	−1	0.184
25.3	28.1	−4.2	2.2	−0.8	−0.8	−0.8	−0.8	−0.102
16.2	23.3	−4.2	3.7	−0.7	−2.1	1.2	−3.5	0.226
23.2	23.9	−1.2	5.7	2.4	1.9	2.6	1.7	0.185
17.9	18	2	9.7	5.4	4.3	6.8	1.8	0.587
23.9	30.5	−7.3	−0.6	−4	−4.3	−3.7	−4.3	−0.269
27.1	24.7	0	6.2	3.5	3.2	3.2	4	0.129
37.5	32.6	−3.6	1.3	−0.3	0.3	0	−4.2	−0.412

Table 8. Comparison of Errors Estimated by SVC and Other Models for Testing Data from HSC

f_c (MPa)	E_c (GPa)	ACI 363	CBE	NS 3473	Wee et al. 1996	Gesoglu et al. 2002	Regression (Demir 2005)	Fuzzy (Demir 2005)	ANN (Demir 2008)	SVM (Yan and Shi 2010)	SVC
69.7	41.5	−7.1	1.2	−7.5	0.4	−0.8	0.4	1.7	2.5	1.7	0.537
78.3	44.3	−8	0	−9.3	−0.4	−0.9	−0.4	0	0.4	−0.9	0.248
82.6	44.2	−7.1	0.9	−8.4	0.4	0.4	0.9	0.9	1.3	0	0.259
65.8	40.8	−0.69	1.2	−7.3	0.4	−1.6	0	1.6	2.4	3.3	0.610
100.6	45.8	−5.5	1.8	−7.8	1.8	4.1	3.2	2.3	1.8	0.9	0.095
92.8	45.8	−6.9	0.9	−8.7	0.5	1.8	1.4	0.9	1.8	0	0.096
93.6	47.1	−8	−0.5	−9.9	−0.9	0.9	0.5	0	0.5	−0.9	−0.038
71.5	48	−13	−4.8	−13.9	−6.7	−5.8	−5.8	−4.8	−3.8	−3.4	−0.132
59.1	40.9	−8.6	−0.4	−8.6	−4.1	−1.2	−1.6	0.4	0.4	3.3	0.599
57.9	44.5	−12.5	−4	−12.5	−8	−4.9	−5.8	−3.1	−3.6	1.8	0.226
93.7	50.5	−11.6	−4	−13.6	−2.5	−4	−3	−3.5	−3	−4	−0.392
85.3	45	−7.7	0.5	−9	0.5	0	0.9	0.5	0.5	0	0.176
99.7	47.6	−7.6	0	−10	1.9	−0.5	1.4	0.5	0	−1	−0.092
85.1	44.7	−7.2	0.4	−8.5	0.9	0	0.9	0.9	0.9	0	0.208
90.3	45	−6.8	1.4	−8.1	2.3	0.9	1.8	1.4	0.9	0.9	0.177
87.2	41.1	−3.3	4.5	−4.9	4.9	4.1	4.9	4.9	4.5	4.1	0.580
84.5	45.3	−7.7	0	−9.5	0	−0.5	0	0	0	−0.9	0.147
77	47.2	−11.3	−3.3	−12.3	−3.8	−4.2	−3.3	−4.7	−2.4	−2.4	−0.049
86	43.8	−6.1	1.8	−7.4	1.3	2.2	2.2	0	1.8	1.3	0.302
86	42.3	−4.7	3	−6.3	2.5	3.4	3.4	1.7	3.4	2.5	0.457

HSC data has desirable accuracy. Tables 2 and 3 show that three and two clusters give the best results for NSC and HSC, respectively. Hence, sensitivity analysis of the α parameter and the kernel (erbf, rbf, and polynomial 2, 3, and 4) on the MAPE and the RMSE for these numbers of clusters are shown in Figs. 6 and 7. In these figures, MAPE and RMSE for the train and test data for the kernel (erbf, rbf, polynomial 2, 3, and 4) and various α parameters have been examined. Results presented in Fig. 6 show that for NSC, the erbf kernel has the least error for the training data and polynomial kernel ($p = 3$ and 4) for the test data at $\alpha = 0.6$. Examination of Fig. 7 for HSC reveals that the erbf kernel has the least error for both train and test data at $\alpha = 35$.

Quantitative Evaluation of SVC Procedure

The role of the α parameter and clustering were explained in the previous section. This section further expands on the evaluation of the membership value (w). To be precise, the membership value of each test data point in the Gaussian membership function obtained from the train data in the cluster is calculated. Details of this process are as follows:

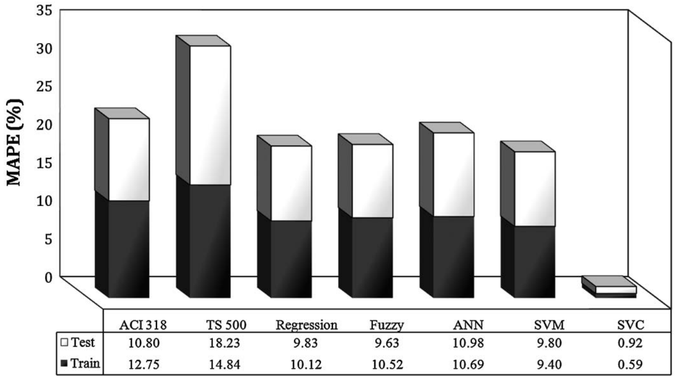

Fig. 10. Comparison between SVC and other models in terms of MAPE for NSC

The NSC data has been subdivided into three clusters: $14 \le f_c \le 24$ MPa, $24 < f_c < 35$ MPa, and $f_c \ge 35$ MPa, whereas the HSC data was split into two clusters of $46 \le f_c \le 80$ and $f_c > 80$.

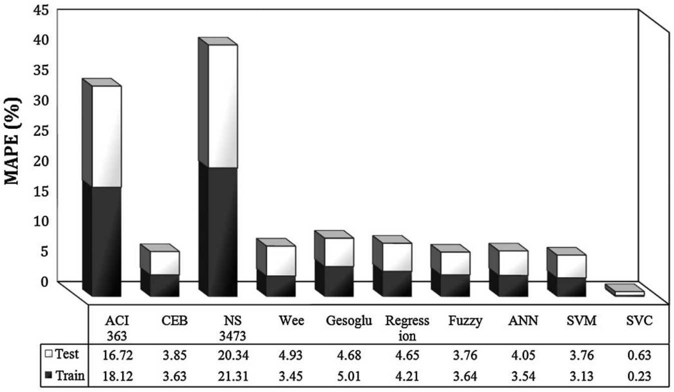

Fig. 11. Comparison between SVC and other models in terms of MAPE for HSC

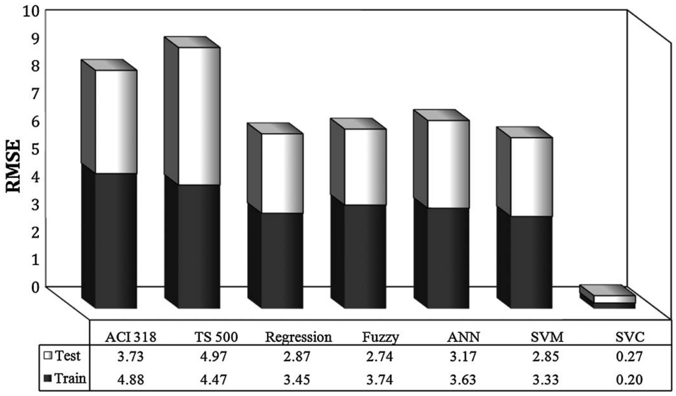

Fig. 12. Comparison between SVC and other models in terms of RMSE for NSC

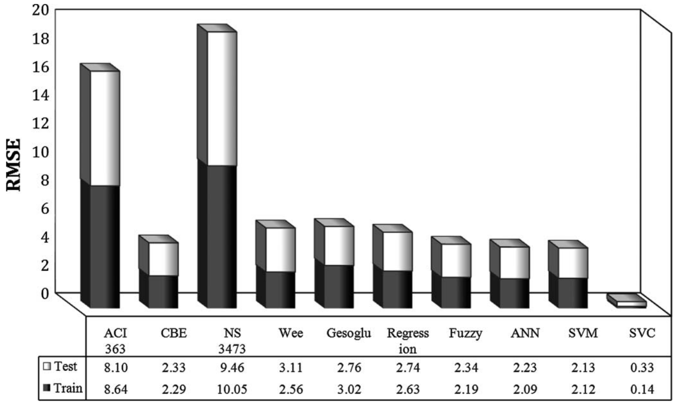

Fig. 13. Comparison between SVC and other models in terms of RMSE for HSC

Table 9. Parameter Appropriate Values Used in SVC

Data type	Kernel function	Cluster	α
NSC	erbf	3	0.6
HSC	erbf	2	35

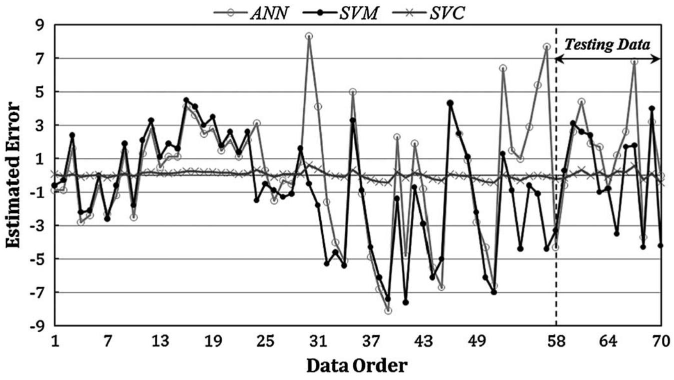

Fig. 14. Comparing error of ANN, SVM, and SVC for NSC data set

A number of data points (15, 35, and 7 for NSC and 18 and 51 for HSC) were used as the training set, and the rest were used as test data. Membership value (w) of each test data point in the Gaussian membership function obtained from the training data of each cluster were calculated using Eq. (9). Details of some of the data points and the clustering are shown in Figs. 8 and 9 for NSC and HSC, respectively.

Finally, statistical information of the membership values for train and test data is presented in Table 4.

Comparison of Calculated Result with Conventional Method

The errors computed by the SVC model, other researchers' models, and experimental results for training and testing data sets for NSC and HSC are shown in Tables 5–8.

Based on the comparison of the results presented in the tables, it may be concluded that the SVC can predict the elastic modulus of NSC and HSC with very high accuracy.

A further comparison between SVC and other available models can be made using MAPE (%) and RMSE for NSC and HSC. The results are shown in Figs. 10–13.

Conclusion

In this paper, the SVC method is used for prediction of elastic modulus of NSC and HSC. The effects of several parameters on experimental data have been studied, and appropriate values of parameters selected are shown in Table 9.

The next step, elastic modulus estimated by SVC, considers appropriate values of parameters, and the difference between estimated and experimental values were calculated. The error was compared with the results of the best predictions to date (i.e., the ANN and SVM methods), and it was shown that marked improvement with respect to the best of other methods may be attained by the SVC method. A representation of the accuracy of the method is shown in Figs. 14 and 15.

Finally, the estimated accuracy is calculated by MAPE and RMSE as error criteria and compared with other researchers' models. The MAPE values for NSC and HSC decreased from 10 and 4%, respectively, to less than 1%. The values of RMSE for NSC and HSC decreased from 3 to less than 0.3.

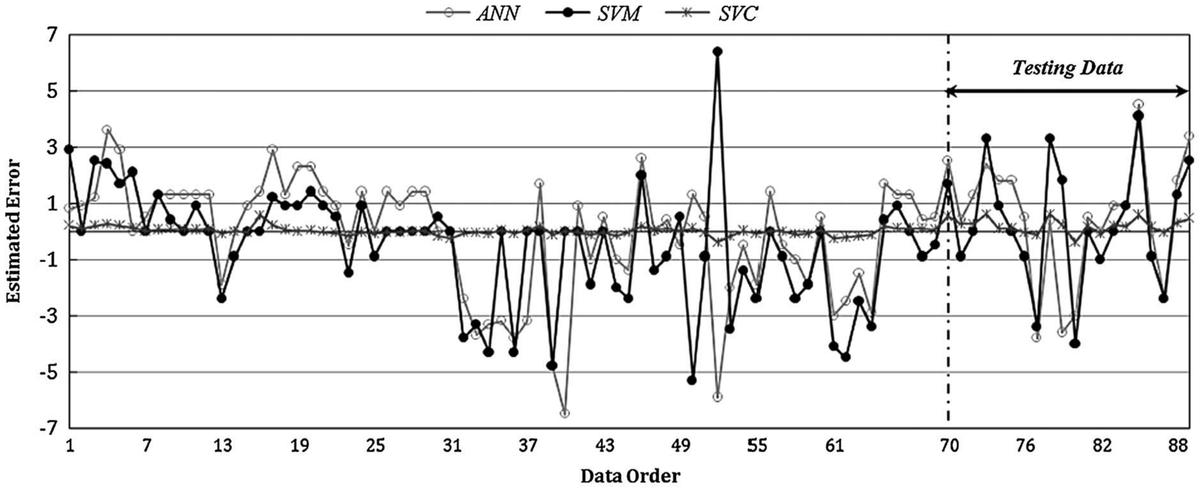

Fig. 15. Comparing error of ANN, SVM, and SVC for HSC data set

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It may therefore be concluded that SVC is the most effective method for prediction of elastic modulus of all grades of concrete from their compressive strength.

Appendix. Fuzzy C-Means

In this section, the conventional fuzzy C-means algorithm (Bezdek 1981) is mentioned, which is one of the most widely used fuzzy clustering methods. The FCM clustering method is the fuzzy equivalent of the nearest mean hard clustering method. The FCM clustering method assigns fuzzy memberships to each input member. The aim of the FCM algorithm is to minimize the following objective function with respect to fuzzy membership, u_{ik}, and cluster centroid, v_i.

$$J_m(U,V) = \sum_{k=1}^{n}\sum_{i=1}^{c} u_{i,k}^m d^2(x_k, v_i) \tag{10}$$

$$d^2(x_k, v_i) = (x_k - v_i)^T(x_k - v_i) = \|x_k - v_i\|^2 \tag{11}$$

where $X = \{x_1, x_2, \ldots, x_n\}$ = set of features vectors and is a finite set of p-dimensional vectors over the real numbers ($x_k = [x_{k,1}, x_{k,2}, \ldots, x_{k,p}]^T$ for, $= 1, \ldots, n$); c = number of clusters; and $m > 1$ = fuzziness index. The matrix $U = [u_{i,k}]_{c\times n}$ = fuzzy membership degree that has following constraint:

$$\begin{aligned} & u_{i,k} \in [0,1], \quad i = 1,2,\ldots,c, \quad k = 1,2,\ldots,n \\ & \sum_{i=1}^{c} u_{i,k} = 1, \quad k = 1,2,\ldots,n \\ & 0 < \sum_{k=1}^{n} u_{i,k} < n, \quad i = 1,2,\ldots,c. \end{aligned} \tag{12}$$

where u_{ik} = membership grade of kth numbers to ith cluster; $V = \{v_1, v_2, \ldots, v_c\}$ = cluster prototypes set; and $v_i = [v_{i,1}, v_{i,2}, \ldots, v_{i,p}]^T \in R^p$, $i = 1,2,\ldots,c$ = center of ith cluster.

By creating Lagrange function, $J_m(U,V)$ can be minimized subject to the constraints in Eq. (12) and conclude updating related as follows:

$$\begin{aligned} L(V,U,\lambda) = & \sum_{k=1}^{n}\sum_{i=1}^{c} u_{i,k}^m (x_k - v_i)^T(x_k - v_i) \\ & - \sum_{k=1}^{n} \lambda_k \left(\sum_{i=1}^{c} u_{i,k} - 1\right), \end{aligned} \tag{13}$$

$$\begin{aligned} & \frac{\partial L}{\partial u_{i,k}} = 0, \qquad \frac{\partial L}{\partial \lambda_k} = 0 \\ & \Rightarrow u_{i,k} = \left(\sum_{j=1}^{c}\left(\frac{d^2(x_k, v_i)}{d^2(x_k, v_j)}\right)^{1/m-1}\right)^{-1} \\ & = \left(\sum_{j=1}^{c}\left(\frac{\|x_k - v_i\|}{\|x_k - v_j\|}\right)^{2/m-1}\right)^{-1}, \\ & \text{for } i = 1,2,\ldots,c \quad \text{and} \quad k = 1,2,\ldots,n \end{aligned} \tag{14}$$

$$\frac{\partial L}{\partial v_i} = 0 \Rightarrow v_i = \frac{\sum_{k=1}^{n}(u_{i,k})^m x_k}{\sum_{k=1}^{n}(u_{i,k})^m}, \quad \text{for } i = 1,2,\ldots,c. \tag{15}$$

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Notation

The following symbols are used in this paper:

c = kernel function constant;
E_c = concrete elastic modulus;
err = difference between predicted and experimented values;
f_c = compressive strength;
K = kernel matrix;
w = energy;
x_t = test sample;
α, α^* = Lagrange coefficient;
ε = insensitive function as the loss function;
μ = mean value of training data;
ξ_i, ξ_i^* = slack variables for soft margin SVR;
σ = width of Gaussian; and
ϕ = nonlinear transformation.

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